On the probability of crystallization

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On the probability of crystallization

Abstract. It is shown that the equations for the probability of crystallization of supercooled aerosol obtained by Carte and Bigg are special cases of the more general equation.

In Carte's (1959) letter it was shown that Bigg's equations could be obtained in a more direct way than was done by Bigg (1953), and three equations equivalent to those of Bigg's were given also.

It should be remembered that both the equations Carte mentions and the technique of their deduction were already given by Kachurin (1953) irrespective of Carte (1959) and Bigg (1953). The equation (4) from the paper by Kachurin (1953) is

$$\frac{W_\Omega}{\eta(\Omega)} = 1 - \exp\left(-\int_{T_0}^{T} w\Omega \, \mathrm{d}T\right). \tag{1}$$

The equation (1) in the most general form from Carte (1959) is

$$P(V, t) = 1 - \exp\left(-\int_0^t Vf(T_\mathrm{s}) \, \mathrm{d}t\right). \tag{2}$$

The parameter Ω is equal to

$$\Omega = \frac{\frac{4}{3}\pi\tau^3}{\mathrm{d}T/\mathrm{d}t} \tag{3}$$

where τ is the radius of the drop, T the temperature and t the time.

The same parameter in Carte's (1959) designation is

$$\Omega = \frac{V}{\mathrm{d}T_\mathrm{s}/\mathrm{d}t}$$

where V is the volume of the drop, T_s the temperature of the drop, W_Ω the relative number of frozen drops of given value Ω and $\eta(\Omega)$ the total number of frozen drops of given value Ω.

If we take equation (3) into account, equation (1) takes the form

$$\frac{W_\Omega}{\eta(\Omega)} = 1 - \exp\left(-\int_0^t \tfrac{4}{3}\pi\tau^3 w \, \mathrm{d}t\right). \tag{4}$$

Thus in the left-hand sides of equations (1) and (4) there is the probability of drop freezing, which is characterized by the value Ω. The same probability is seen in the left-hand side of equation (2).

It should be noted that the expression $P(V, t)$ is not quite exact, for the value P depends on the drop volume V and the time t at a constant rate of cooling only, but Bigg's equation (1) is true only for a general case; a special equation is written for the constant cooling rate in the paper by Carte (1959).

The probability of a heterophasic ice nucleation rate attributed to the volume unit is given by Kachurin (1953) as $w(f(T_\mathrm{s}) \equiv w)$, and by Carte (1959) as $f(T_\mathrm{s})$. Thus it can be seen that the above-mentioned equations are identical.

A more general equation is presented by Kachurin (1953), one of the variants of which is the above-mentioned equation (1). This general equation is expressed as

$$W = 1 - \int_{0}^{-\infty} \eta(\Omega) \exp\left(-\int_{T_0}^{T} w\Omega \, \mathrm{d}T\right) \mathrm{d}\Omega \tag{5}$$

and corresponds to the case when the polydispersional aerosol is crystallized, all its drop sizes being characterized by its own cooling rate.

Equations (3) were repeatedly used to calculate the crystallization of the aerosol (see, for example, Kachurin 1959).

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Note on an interesting metric of the field equations in general relativity

Abstract. In the course of investigating the Rainich equations of the 'already unified field theory' in the case when the electromagnetic field is non-static, Bera and Datta in 1968 obtained a metric which leads to empty flat space. It is shown that the said metric is Riemannian (non-flat) and satisfies the field equations of gravitation for empty space. Hence it is of great interest from the point of view of cosmology.

In the course of investigating the Rainich equations of the 'already unified field theory' in the case when the electromagnetic field is non-static and the space–time metric admits a group G_4 of automorphisms, Bera and Datta (1968) have very recently obtained the metric

$$\mathrm{d}s^2 = (\mathrm{d}x^4)^2 - B(kx^4+l)^{4/3}\{(\mathrm{d}x^1)^2+(\mathrm{d}x^2)^2\} - \frac{16AB}{9}(kx^4+l)^{-2/3}(\mathrm{d}x^3)^2 \tag{1}$$

for which

$$R_4{}^4 = 0. \tag{2}$$

Here x^4 is the time coordinate and x^1, x^2, x^3 are the space coordinates.

We note in passing that the metric (1) admits an intransitive group of motions and that the group G_4 includes the Abelian subgroups G_3.

The metric is seen to satisfy the field equations of gravitation for empty space

$$G_{\mu\nu} = 0 \tag{3}$$

and, moreover, the pseudo-tensor density of gravitational energy and momentum vanishes everywhere, i.e.

$$t_\mu{}^\nu = 0. \tag{4}$$